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THE STRESSED STATE NEAR CURVILINEAR REINFORCED ORIFICES IN SHELLS

A. N. Guz' and G. N. Savin

ABSTRACT

Investigation of the stressed state in shells near curvilinear orifices reinforced with thin elastic rings. The general case is considered, as well as the particular case of the stressed state of a spherical shell loaded with a uniform internal pressure and weakened by an elliptical orifice with a small eccentricity, the edge of this orifice being reinforced with an elastic ring.

In reference 1 an approximate method was proposed for investigating the state of stress in shells weakened by curvilinear holes whose contours had no angular points. In the present paper, proceeding from the general formulation of the problem given in reference 2 and the results of reference 1, we investigate the state of stress in shells near curvilinear holes supported by thin elastic rings (considered as material threads) which resist tension, flexure and torsion.[1] The boundary values for this case were obtained in reference 3. /103*

1. The investigation of the additional state of stress in the shell near a curvilinear reinforced hole is reduced (refs. 2, 1) to the solution of the equation

$$\nabla^2\nabla^2\Phi - i\varkappa^2\nabla_k^2\Phi = 0 \tag{1.1}$$

with the corresponding boundary conditions. Equation (1.1) is written in dimensionless coordinates referred to r_0. Assuming that one of the axes of inertia for the transverse cross-section of the supported ring lies in the median surface of the shell and assuming the basic assumptions of the theory of hollow shells, the boundary conditions can be written in the form (ref. 3)

*Numbers given in margin indicate pagination in original foreign text.
[1]We use the designations adopted in reference 1.

$$T_n + T_n^0 = T_n^{(0)} - B\frac{\partial^3}{\partial s^3}\left[\frac{u_s + u_s^0}{R^*} - \frac{\partial}{\partial s}(u_n + u_n^0)\right] +$$
$$+ \frac{E_1 F}{R^*}\left[\frac{\partial}{\partial s}(u_s + u_s^0) + \frac{u_n + u_n^0}{R^*}\right],$$
$$S_{ns} + S_{ns}^0 = S_{ns}^{(0)} - \frac{B}{R^*}\frac{\partial^2}{\partial s^2}\left[\frac{u_s + u_s^0}{R^*} - \frac{\partial}{\partial s}(u_n + u_n^0)\right] -$$
$$- E_1 F\frac{\partial}{\partial s}\left[\frac{\partial}{\partial s}(u_s + u_s^0) + \frac{u_n + u_n^0}{R^*}\right], \tag{1.2}$$
$$G_n + G_n^0 = G_n^{(0)} + \left[C\frac{\partial}{\partial s}\left(\frac{\partial^2}{\partial n\,\partial s} - \frac{1}{R^*}\frac{\partial}{\partial s}\right) - \frac{A}{R^*}\left(\frac{1}{R^*}\frac{\partial}{\partial n} + \frac{\partial^2}{\partial s^2}\right)\right](w + w^0)$$
$$\tilde{Q}_n + \tilde{Q}_n^0 = \tilde{Q}_n^{(0)} + \frac{\partial}{\partial s}\left[\frac{C}{R^*}\left(\frac{\partial^2}{\partial n\,\partial s} - \frac{1}{R^*}\frac{\partial}{\partial s}\right) + A\frac{\partial}{\partial s}\left(\frac{1}{R^*}\frac{\partial}{\partial n} + \right.\right.$$
$$\left.\left. + \frac{\partial^2}{\partial s^2}\right)\right](w + w^0),$$

where $(T_n, \ldots, \tilde{Q}_n, u_n, u_s, w)$ are the components of the additional stresses and strains; $(T_n^0, \ldots, \tilde{Q}_n^0, u_n^0, u_s^0, w^0)$ are the components of the basic stresses and strains;[1] $(T_n^{(0)}, \ldots, \tilde{Q}_n^{(0)})$ are the external loads acting on the sup- /104 porting ring; R^* is the radius of curvature of the hole contour in the plane of variables associated with the median surface of the shell; A, B and C is the flexural rigidity of the ring with respect to two axes and the torsional rigidity. F is the cross-section area of the ring.

Let us assume that the function

$$z = \zeta + \varepsilon f(\zeta) \quad (z = re^{i\theta},\ z = re^{i\gamma}) \tag{1.3}$$

in the plane of the variables associated with the median surface of the shell produces a conformal transformation of an infinite plane with a circular hole of unit radius into an infinite plane with the considered hole.[2] Following reference 1, we represent all the quantities in (1.1) and (1.2) as expansions in series of ϵ. Substituting these quantities into (1.2) and collecting the coefficients in front of ϵ^j, we obtain the boundary conditions in the j-th approximation

[1]We use the terminology adopted in reference 2.
[2]We should note that when $\epsilon \ll 1$ the shape of the hole contour may vary substantially from that of a circle.

$$T_n^{(j)}|_{\rho=1} + T_n^{0(j)}|_{\rho=1} = T_n^{(0)(j)}|_{\rho=1} - \frac{B}{r_0^4\rho^4}\frac{\partial^3}{\partial\gamma^3}\Big[v^{(j)} + \delta_j^0 v^0 -$$
$$-\frac{\partial}{\partial\gamma}(u^{(j)} + \delta_j^0 u^0)\Big]\Big|_{\rho=1} + \frac{E_1F}{r_0^2\rho^2}\Big[\frac{\partial}{\partial\gamma}(v^{(j)} + \delta_j^0 v^0)\Big] +$$
$$+ u^{(j)} + \delta_j^0 u^0\Big]\Big|_{\rho=1} + \sum_{m=0}^{j-1}[L_7^{(j-m)}(u^{(m)} + \delta_m^0 u^0) + L_8^{(j-m)}(v^{(m)} + \delta_m^0 v^0)]|_{\rho=1},$$
$$S_{ns}^{(j)}|_{\rho=1} + S_{ns}^{0(j)}|_{\rho=1} = S_{ns}^{(0)(j)}|_{\rho=1} - \frac{B}{r_0^4\rho^4}\frac{\partial^2}{\partial\gamma^2}\Big[v^{(j)} + \delta_j^0 v^0 -$$
$$-\frac{\partial}{\partial\gamma}(u^{(j)} + \delta_j^0 u^0)\Big]\Big|_{\rho=1} - \frac{E_1E}{r_0^2\rho^2}\frac{\partial}{\partial\gamma}\Big[\frac{\partial}{\partial\gamma}(v^{(j)} + \delta_j^0 v^0) + u^{(j)} + \delta_j^0 u^0\Big]\Big|_{\rho=1} +$$
$$+ \sum_{m=0}^{j-1}[L_9^{(j-m)}(u^{(m)} + \delta_m^0 u^0) + L_{10}^{(j-m)}(v^{(m)} + \delta_m^0 v^0)]|_{\rho=1},$$
$$G_n^{(j)}|_{\rho=1} + G_n^{0(j)}|_{\rho=1} = G_n^{(0)(j)}|_{\rho=1} + \frac{1}{r_0^3}\Big[\frac{C}{\rho}\frac{\partial^3}{\partial\rho\,\partial\gamma^2}\frac{1}{\rho} - \frac{A}{\rho^2}\Big(\frac{\partial}{\partial\rho} + \qquad (1.4)$$
$$+\frac{1}{\rho}\frac{\partial^2}{\partial\gamma^2}\Big)\Big][\mathrm{Re}\,\Phi_j(\rho,\gamma) + \delta_j^0 w^0]|_{\rho=1} +$$
$$+\frac{1}{r_0^3}\sum_{m=0}^{j-1}L_{11}^{(j-m)}[\mathrm{Re}\,\Phi_m(\rho,\gamma) + \delta_m^0 w^0)]|_{\rho=1},$$
$$\tilde{Q}_n^{(j)}|_{\rho=1} + \tilde{Q}_n^{0(j)}|_{\rho=1} = \tilde{Q}_n^{(0)(j)}|_{\rho=1} + \frac{1}{r_0^4}\frac{1}{\rho^3}\frac{\partial^2}{\partial\gamma^2}\Big[C\rho\frac{\partial}{\partial\rho}\frac{1}{\rho} +$$
$$+A\Big(\frac{\partial}{\partial\rho} + \frac{1}{\rho}\frac{\partial^2}{\partial\gamma^2}\Big)\Big][\mathrm{Re}\,\Phi_j(\rho,\gamma) + \delta_j^0 w^0]|_{\rho=1} +$$
$$+\frac{1}{r_0^4}\sum_{m=0}^{j-1}L_{12}^{(j-m)}[\mathrm{Re}\,\Phi_m(\rho,\gamma) + \delta_m^0 w^0]|_{\rho=1},$$

where $T_n^{0(j)}$, ..., $\tilde{Q}_n^{0(j)}$ are the expansion coefficients of the components of the basic state of stress, ($T_n^{(0)(j)}$, ..., $\tilde{Q}_n^{(0)(j)}$) are the expansion coeffi- /105 cients for external forces which act on the ring, u^0, v^0 and w^0 are the components of the basic state of strain in the polar system of coordinates, in which r and θ are replaced respectively by ρ and γ, $L_7^{(j-m)}$, ..., $L_{12}^{(j-m)}$ are the differential operators whose form is complex and will not be presented here[1] ($L_7^{(0)} \equiv \ldots \equiv L_{12}^{(0)} \equiv 0$); the equations and formulas for determining $\Phi_j(\rho, \gamma)$, $T_n^{(j)}$, ..., $\tilde{Q}_n^{(j)}$, $u^{(j)}$ and $v^{(j)}$ are presented in reference 1.

[1]The operators $L_7^{(1)}$, ..., $L_{12}^{(1)}$ are obtained in the general form. The calculations necessary to obtain the operators for the subsequent approximations are extremely cumbersome.

$$\delta_k^n = \begin{cases} 1 & k = n, \\ 0 & k \neq n. \end{cases}$$

To determine the stress components in the shell it is necessary to make use of results presented in reference 1. From (1.4) and from the equations for Φ_j, $u^{(j)}$ and $v^{(j)}$, we see that in each approximation (with respect to Φ_j) the problem is reduced to the solution of a boundary value problem for a circular hole in the plane ζ.

2. Let us consider the state of stress in a spherical shell loaded with uniform internal pressure p and weakened by an elliptical hole with small eccentricity and with the edges supported by an elastic ring.

For an elliptical hole we have

$$r_0 = \frac{a+b}{2}, \quad \varepsilon = \frac{a-b}{a+b}, \quad f(\zeta) = \frac{1}{\zeta}, \tag{2.1}$$

where a and b are semiaxes of the ellipse.

We shall assume that the basic state of stress of the shell is momentless; then

$$\begin{gathered} T_n^0 = p_0 h, \; T_s^0 = p_0 h, \; S_{ns}^0 = 0, \; G_n^0 = 0, \; \widetilde{Q}_n^0 = 0, \\ u^0 = p_0 \frac{1-\nu}{E} r_0 r, \; v^0 = 0, \quad w^0 = 0, \; p_0 = \frac{pR}{2h}. \end{gathered} \tag{2.2}$$

We shall assume that the hole is closed with a cover of special construction, which transmits the pressure to the ring as a transverse force. We shall assume that the variation in the transverse force along the contour has the form

$$\widetilde{Q}_n^{(0)(0)} = -\frac{pr_0}{2}, \; \widetilde{Q}_n^{(0)(1)} = pr_0 \cos 2\gamma, \ldots \tag{2.3}$$

We shall represent the function Φ (r, θ) (ref. 1) in the form

$$\begin{gathered} \Phi_j(r, \theta) = id_2^{j,0} \ln r + (c_4^{j,0} + id_4^{j,0}) H_0^{(1)}(r\varkappa\sqrt{-i}) + \\ + \sum_{k=1}^{\infty} [(c_2^{j,2k} + ia_2^{j,2k}) r^{-2k} + (c_4^{j,2k} + id_4^{j,2k}) H_2^{(1)}(r\varkappa\sqrt{-i})] \cos 2k\theta, \end{gathered} \tag{2.4}$$

where

$$\varkappa = r_0/\sqrt{Rh}\sqrt[4]{12(1-\nu^2)}, \quad H_{2k}^{(1)}(r\varkappa\sqrt{-i}) = \mathrm{her}_{2k}\,\varkappa r + i\,\mathrm{hei}_{2k}\,\varkappa r.$$

The displacements $u^{(j)}(\rho, \gamma)$ and $v^{(j)}(\rho, \gamma)$ are determined from the system of equations presented in reference 1. Substituting (2.2)-(2.4) into (1.4) we obtain a system of algebraic equations for the coefficients $c_2^{j,\,2k}$, /106 $d_2^{j,\,2k}$, $c_4^{j,\,2k}$ and $d_4^{j,\,2k}$. In the zero approximation we arrive at the corresponding problem with a circular hole of radius r_0, specifically: to the problem on the stress concentration near a circular hole in a spherical shell which is subjected to a uniform internal pressure p. The constants for the zero approximation will be

$$c_4^{0,0} = -\frac{pr_0^4}{2D\varkappa^3}\,\frac{r_0\varkappa D\,\mathrm{hei}''\varkappa + (Dr_0\nu - A)\,\mathrm{hei}'\varkappa}{r_0\varkappa D\,(\mathrm{her}'\varkappa\,\mathrm{her}'\varkappa + \mathrm{hei}'\varkappa\,\mathrm{hei}''\varkappa) + (Dr_0\nu - \theta)(\mathrm{her}'^2\varkappa + \mathrm{hei}'^2\varkappa)},$$

$$d_2^{00} = \frac{pRE_1Fr_0^2\sqrt{12(1-\nu^2)}}{Eh^2[Ehr_0 + E_1F(1+\nu)]}, \tag{2.5}$$

$$d_4^{00} = -\frac{pr_0^4}{2D\varkappa^3}\,\frac{r_0\varkappa D\,\mathrm{her}''\varkappa + (Dr_0\nu - A)\,\mathrm{her}'\varkappa}{r_0\varkappa D\,(\mathrm{her}'\varkappa\,\mathrm{her}''\varkappa + \mathrm{hei}'\varkappa\,\mathrm{hei}''\varkappa) + (Dr_0\nu - A)(\mathrm{her}'^2\varkappa + \mathrm{hei}'^2\varkappa)}.$$

Taking into account reference 1, we obtain the following expression for the components of the state of stress along the contour of the hole ($\rho = 1$)

$$T_n^{*(0)} = p_0h\,\frac{2E_1F}{Ehr_0 + E_1F(1+\nu)}, \quad T_s^{*(0)} = 2p_0h\,\frac{Ehr_0 + \nu E_1F}{Ehr_0 + E_1F(1+\nu)} -$$

$$- p_0h\varkappa\,\frac{r_0\varkappa D\,(\mathrm{hei}'\varkappa\,\mathrm{her}\,\varkappa - \mathrm{hei}\,\varkappa\,\mathrm{her}''\varkappa) + (Dr_0\nu - A)(\mathrm{hei}'\varkappa\,\mathrm{her}\,\varkappa - \mathrm{her}'\varkappa\,\mathrm{hei}\,\varkappa)}{r_0\varkappa D\,(\mathrm{her}'\varkappa\,\mathrm{her}''\varkappa - \mathrm{hei}'\varkappa\,\mathrm{hei}''\varkappa) + (Dr_0\nu - A)(\mathrm{her}'^2\varkappa + \mathrm{hei}'^2\varkappa)},$$

$$G_n^{*(0)} = -p_0h\,\frac{\varkappa hA}{\sqrt{12(1-\nu^2)}} \times \tag{2.6}$$

$$\times\,\frac{\mathrm{hei}'\varkappa\,\mathrm{her}''\varkappa - \mathrm{her}'\varkappa\,\mathrm{hei}''\varkappa}{r_0\varkappa D\,(\mathrm{her}'\varkappa\,\mathrm{her}''\varkappa + \mathrm{hei}'\varkappa\,\mathrm{hei}''\varkappa) + (Dr_0\nu - A)(\mathrm{her}'^2\varkappa + \mathrm{hei}'^2\varkappa)},$$

$$G_s^{*(0)} = p_0h\,\frac{\varkappa h}{\sqrt{12(1-\nu^2)}} \times$$

$$\times\,\frac{(Dr_0 - Dr_0\nu^2 + A\nu)(\mathrm{her}\,\varkappa\,\mathrm{her}'\varkappa + \mathrm{hei}\,\varkappa\,\mathrm{hei}'\varkappa)}{r_0\varkappa D\,(\mathrm{her}'\varkappa\,\mathrm{her}''\varkappa + \mathrm{hei}'\varkappa\,\mathrm{hei}''\varkappa) + (Dr_0\nu - A)(\mathrm{her}'^2\varkappa + \mathrm{hei}'^2\varkappa)}.$$

We note that $T_n^*|_{\rho=1}$ (2.6) for the shell in the case of a circular hole does not depend on the sphere radius R and coincides completely (ref. 4) with $T_n^*|_{\rho=1}$ for a circular hole in a plate subjected to a tension p_0h from all sides. If we let $R = \infty$ in (2.6), we obtain (ref. 4) the values of the stress components for a flat plate; if we let $E_1 = 0$ or $E_1 = \infty$ (in the expressions for the rigidities A, B and C, E_1 is contained as a multiplier), we obtain the values of the stress components on the contour of a circular hole in a spherical shell (ref. 5), which is, respectively, not supported or supported with an absolutely rigid ring.

For a spherical shell with a radius R = 200 cm, a thickness of h = 0.2 cm, r_0 = 10 cm, $\nu = 0.3$, $\nu_1 = 0.3$ and for a ring of square cross-section whose sides are 0.1 r_0 long, figure 1 shows the relationships ($\rho = 1$): T_n^*/p_0h curve I' for the shell and the plate; T_s^*/p_0h curve II' for the shell; curve II for the plate; $6G_n^*/p_0h^2$ curve I", $6G_s^*/p_0h^2$ curve II" as a function of the parameter $\alpha = E_1/(E_1 + E)$.

The system of algebraic equations which is used to determine the constants $C_2^{1,2}$, $d_2^{1,2}$, $C_4^{1,2}$ and $d_4^{1,2}$ when j = 1 is not presented because it is very bulky. /107

The remaining constants in (2.4) are equal to 0 when j = 1.

For small values of ϵ (2.1) we limit ourselves to the zero and to the first approximation. Taking into account the results obtained in reference 1, we write down the expression for the values of the stress components with an accuracy up to ϵ:

$$T_n^* = p_0h + \frac{1}{nr_0^2}\left(\frac{d_2^{0,0}}{\rho^2} + \varkappa\,\frac{c_4^{0,0}\,\mathrm{hei}'\,\varkappa\rho + d_4^{0,0}\,\mathrm{her}'\,\varkappa\rho}{\rho}\right) + \varepsilon\,\frac{\varkappa^2}{nr_0^2}\left[-6\,\frac{d_2^{1,2}}{\varkappa^2\rho^4} + \right.$$

$$+ c_4^{1,2}\left(\mathrm{her}_2\varkappa\rho - 4\,\mathrm{hei}_2''\,\varkappa\rho\right) - d_4^{1,2}\left(\mathrm{hei}_2\,\varkappa\rho + 4\,\mathrm{her}_2''\,\varkappa\rho\right) - \frac{2}{\varkappa^2}\,\frac{d_2^{0,0}}{\rho^4} +$$

$$\left. + c_4^{0,0}\left(\frac{\mathrm{hei}''\,\varkappa\rho}{\rho^2} - \frac{\mathrm{hei}'\,\varkappa\rho}{\varkappa\rho^3}\right) + d_4^{0,0}\left(\frac{\mathrm{her}''\,\varkappa\rho}{\rho^2} - \frac{\mathrm{her}'\,\varkappa\rho}{\varkappa\rho^3}\right)\right]\cos 2\gamma,$$

$$T_s^* = p_0h + \frac{1}{nr_0^2}\left(-\frac{d_2^{0,0}}{\rho^2} + c_4^{0,0}\,\varkappa^2\,\mathrm{hei}''\,\varkappa\rho + d_4^{0,0}\varkappa^2\,\mathrm{her}''\,\varkappa\rho\right) +$$

$$+ \varepsilon\,\frac{\varkappa^2}{nr_0^2}\left[6\,\frac{d_2^{1,2}}{\varkappa^2\rho^4} + c_4^{1,2}\,\mathrm{hei}_2''\,\varkappa\rho + d_4^{1,2}\,\mathrm{her}_2''\,\varkappa\rho + 2\,\frac{d^{0,0}}{\varkappa^2\rho^4} + \right.$$

$$\left. + c_4^{0,0}\varkappa\,\frac{\mathrm{hei}'''\varkappa\rho}{\rho} + d_4^{0,0}\varkappa\,\frac{\mathrm{her}'''\,\varkappa\rho}{\rho}\right]\cos 2\gamma,$$

$$G_n^* = -D\,\frac{\varkappa^2}{r_0^2}\{c_4^{0,0}\,[(1-\nu)\,\mathrm{her}''\,\varkappa\rho - \nu\,\mathrm{hei}\,\varkappa\rho] - d_4^{0,0}\,[(1-\nu)\,\mathrm{hei}''\,\varkappa\rho +$$

$$+ \nu\,\mathrm{her}\,\varkappa\rho]\} - \varepsilon D\,\frac{\varkappa^2}{r_0^2}\left\{6c_2^{1,2}\,\frac{1-\nu}{\varkappa^2\rho^4} + c_4^{1,2}\,\left[(1-\nu)\,\mathrm{her}_2''\,\varkappa\rho - \right.\right.$$

$$- \nu \,\mathrm{hei}_2\, \varkappa\rho] - d_4^{1,2} [(1-\nu)\,\mathrm{hei}_2''\, \varkappa\rho + \nu \,\mathrm{her}_2\, \varkappa\rho] +$$

$$+ \frac{\varkappa}{\rho} c_4^{0,0} [(1-\nu)\,\mathrm{her}'''\, \varkappa\rho - \nu \,\mathrm{hei}'\, \varkappa\rho] - d_4^{0,0} \frac{\varkappa}{\rho} [(1-\nu)\,\mathrm{hei}'''\, \varkappa\rho +$$

$$+ \nu \,\mathrm{her}'\, \varkappa\rho]\Big\} \cos 2\gamma,$$

$$G_s^* = D \frac{\varkappa^2}{r_0^2} \{c_4^{00} [\mathrm{hei}\, \varkappa\rho + (1-\nu)\,\mathrm{her}''\, \varkappa\rho] + d_4^{00} [\mathrm{her}\, \varkappa\rho - (1-\nu)\,\mathrm{hei}''\, \varkappa\rho]\} +$$

$$+ \varepsilon D \frac{\varkappa^2}{\rho^r} \frac{\varkappa^2}{\rho^2} \Big\{ c_2^{1,2} \frac{1-\nu}{\varkappa^2\rho^4} + c_4^{1,2} [(1-\nu)\,\mathrm{her}_2''\, \varkappa\rho + \mathrm{hei}_2\, \varkappa\rho] + d_4^{1,2} [-(1-\nu)\,\mathrm{hei}_2''\, \varkappa\rho +$$

$$+ \mathrm{her}_2\, \varkappa\rho] + \frac{\varkappa}{\rho} c_4^{0,0} [\mathrm{hei}'\, \varkappa\rho + (1-\nu)\,\mathrm{her}'''\, \varkappa\rho] +$$

$$+ \frac{\varkappa}{\rho} d_4^{0,0} [\mathrm{her}'\, \varkappa\rho - (1-\nu)\,\mathrm{hei}'''\, \varkappa\rho]\Big\} \cos 2\gamma.$$

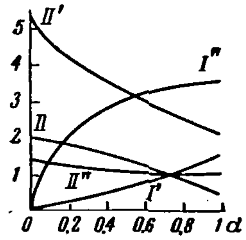

Figure 1

To obtain stress-state value components for the hole contour, we must have $\rho = 1$ in (2.7). The components T_n^*, T_s^*, G_n^* and G_s^* along the sections $\gamma = 0$ and $\gamma = \pi/2$ with $E_1 = 0$; $E_1/E = 2.632$; $E_1 = \infty$ were determined for a shell and a ring with the following parameters: $R = 200$ cm, $r_0 = 10$ cm; $h = 0.2$ cm; $\nu = 0.3$; $\nu_1 = 0.3$ (ν_1 for the ring); $a/b = 1.2$; a ring of square cross-section with a side length of $0.1\, r_0$.

Figures 2-5 show the variation, respectively, in T_n^*/p_0h, T_s^*/p_0h, $6G_n^*/p_0h^2$ and $6G_s^*/p_0h^2$ computed by means of equation (2.7) as a function of the dimensionless parameter ι--the distance from the contour of the hole with respect to $r_0 = (a + b)/2$. The solid line pertains to a circular hole, while the broken line pertains to an elliptical hole with $\gamma = 0$. The dot-dash line pertains to elliptical hole with $\gamma = \pi/2$, and the curves designated by index I correspond to the value $E_1 = \infty$, those with index II correspond to $E_1/E = 2.632$, while those with index III correspond to $E_1 = 0$.

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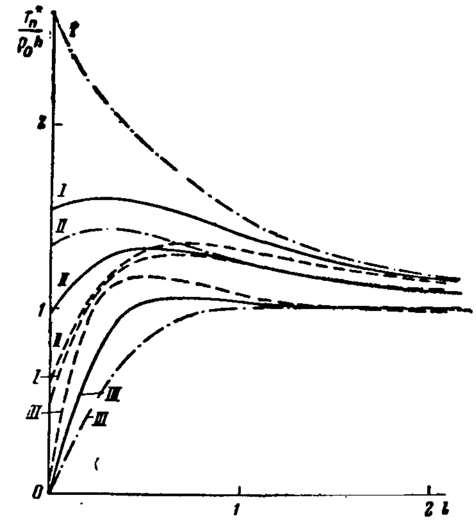

Figure 2

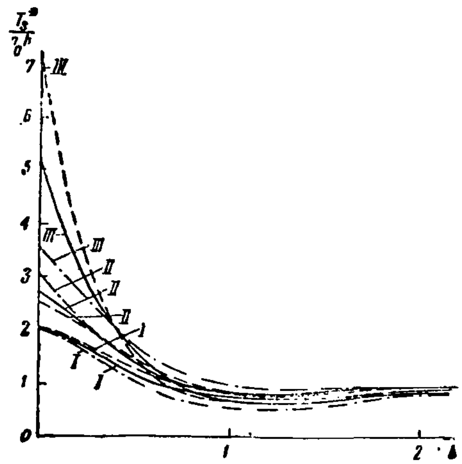

Figure 3

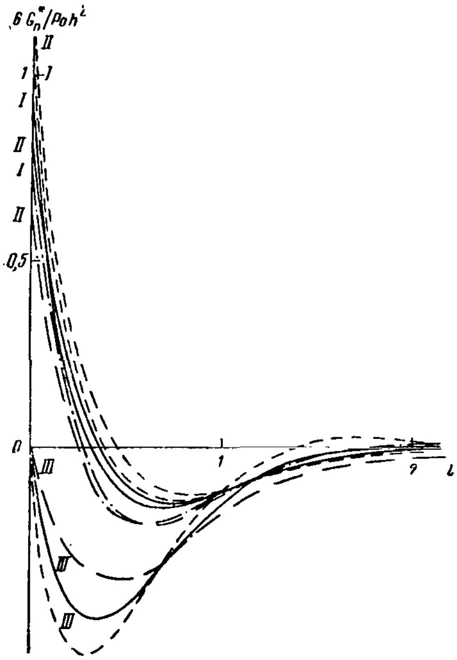

Figure 4

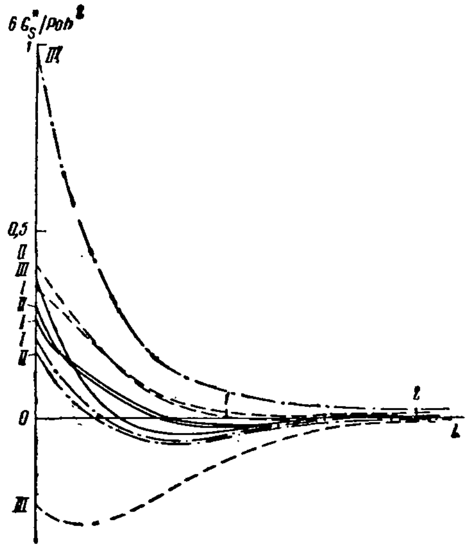

Figure 5

We can see from the graphs that even a small elliptical hole ($a/b = 1.2$) has a strong effect on the stress distribution near it. When we move away a distance of $1.5 r_0 - 2 r_0$ from the hole, the distribution of forces and moments near the hole in the shell is very close to that of the state of stress in a circular hole, and as we move further away it approaches a momentless state of stress. As the rigidity of the supporting ring increases, the concentration of forces and moments increases at the end of the minor semiaxis and at the end of the major semiaxis.

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